September 1998

How to Value and Hedge Options on Foreign Indexes

Kresimir Demeterfi
This material is for your private information, and we are not soliciting any action based upon it. This report is not to be construed as an offer to sell or the solicitation of an offer to buy any security in any jurisdiction where such an offer or solicitation would be illegal. Certain transactions, including those involving futures, options and high yield securities, give rise to substantial risk and are not suitable for all investors. Opinions expressed are our present opinions only. The material is based upon information that we consider reliable, but we do not represent that it is accurate or complete, and it should not be relied upon as such. We, our affiliates, or persons involved in the preparation or issuance of this material, may from time to time have long or short positions and buy or sell securities, futures or options identical with or related to those mentioned herein.

This material has been issued by Goldman, Sachs & Co. and/or one of its affiliates and has been approved by Goldman Sachs International, regulated by The Securities and Futures Authority, in connection with its distribution in the United Kingdom and by Goldman Sachs Canada in connection with its distribution in Canada. This material is distributed in Hong Kong by Goldman Sachs (Asia) L.L.C., and in Japan by Goldman Sachs (Japan) Ltd. This material is not for distribution to private customers, as defined by the rules of The Securities and Futures Authority in the United Kingdom, and any investments including any convertible bonds or derivatives mentioned in this material will not be made available by us to any such private customer. Neither Goldman, Sachs & Co. nor its representative in Seoul, Korea is licensed to engage in securities business in the Republic of Korea. Goldman Sachs International or its affiliates may have acted upon or used this research prior to or immediately following its publication. Foreign currency denominated securities are subject to fluctuations in exchange rates that could have an adverse effect on the value or price of or income derived from the investment. Further information on any of the securities mentioned in this material may be obtained upon request and for this purpose persons in Italy should contact Goldman Sachs S.p.A. in Milan, or at its London branch office at 133 Fleet Street, and persons in Hong Kong should contact Goldman Sachs Asia L.L.C. at 3 Garden Road. Unless governing law permits otherwise, you must contact a Goldman Sachs entity in your home jurisdiction if you want to use our services in effecting a transaction in the securities mentioned in this material.

Note: Options are not suitable for all investors. Please ensure that you have read and understood the current options disclosure document before entering into any options transactions.
SUMMARY

Options on foreign stocks come in several styles, each with a different exposure to foreign currency. Each contract's value and hedge ratio depends, in a specific way, upon the stock volatility, the currency volatility, and the foreign and domestic interest rates. In this note, we review the different options, explain how to value and hedge them, and describe their risks.

Kresimir Demeterfi  (212) 357-4611

I am grateful to Emanuel Derman, Michael Kamal and Joe Zou for comments on the manuscript and to Barbara Dunn for editorial help.
INTRODUCTION

Investors and traders increasingly use derivatives on foreign indexes to obtain exposure to global markets. There are a variety of option styles available, each of which provides different degrees of exposure to the currency of the foreign index. In this note, we provide a summary of the instruments available, and explain how to value and hedge them. To be specific, many of our illustrations will refer to U.S. dollar-based investors who participate in Japanese equity markets, but measure their profits and losses in U.S. dollars.

There are three different types of derivative instruments, each having a different exposure to the foreign stock or index level (say, the Nikkei 225) and the foreign currency (the yen).

1. Foreign-Market Derivatives.
   This is the simplest case: the investor buys a yen-denominated derivative on a Japanese stock, and pays for it today by converting his dollars to yen at the current exchange rate. The investor is exposed to the current stock value in yen (which affects the derivative security’s value in yen) and the value of the yen (which affects the option’s value in dollars). An example is an investor who buys yen-denominated calls on the Nikkei, seeing great potential in the Japanese equity market and expecting the yen to remain stable or even appreciate against the dollar.

2. ADR-style/Composite Derivatives.
   Here, the derivative’s underlyer is the value of the Japanese stock converted to U.S. dollars at the prevailing exchange rate. The investor is exposed to the current American Depositary Receipt value of the stock in dollars, composed of the product of its value in yen and the current value of the yen in dollars. Consequently, such derivatives are sometimes called composite derivatives.

3. GER-style/Quanto Derivatives.
   In this case, the derivative contract provides U.S. dollar exposure to Japanese stock returns with no currency risk at all. The payoff of the derivative depends only on the change in the yen value of the Japanese stock, converted to dollars at a Guaranteed Exchange Rate agreed upon at the inception of the contract.

Each of the three contracts has a value and hedge ratio that depends in different ways on the respective volatilities of the foreign stock and the currency. Similarly, each contract’s value depends in different ways on the foreign and domestic interest rates. In this note, we elaborate on these contracts and explain how to value and hedge them.
For illustrative purposes, we refer to U.S. dollar-based investors who participate in Japanese equity index markets, but measure their profits and losses in U.S. dollars. We use the following notation:

- \( t \): current time
- \( T \): delivery time of forward contract or expiration time of option
- \( S \): index value in yen at time \( t \)
- \( S_T \): index value in yen at expiration
- \( q \): continuous dividend rate on index
- \( X \): dollar value of one yen at time \( t \)
- \( X_T \): dollar value of one yen at expiration
- \( X_0 \): guaranteed exchange rate in dollars per yen
- \( K \): delivery or strike price in yen
- \( K_S \): delivery or strike price in dollars
- \( f(S, K) \): current value of a forward contract to deliver the index at expiration for delivery price \( K \)
- \( F_S \): forward price or the fair delivery value of forward contract on foreign index in yen
- \( F_X \): forward price or the fair delivery value of forward contract on yen in dollars
- \( r_S \): U.S. riskless interest rate
- \( l_S \): U.S. stock loan rate
- \( r_Y \): Japanese riskless interest rate
- \( l_Y \): Japanese stock loan rate
- \( b \): U.S. or Japanese rebate rate earned by lending the stock
- \( \sigma_S \): estimated return volatility of index in yen
- \( \sigma_X \): estimated return volatility of yen in dollars
- \( \sigma_{XS} \): estimated covariance between returns on \( X \) and \( S \)
- \( \rho_{XS} \): estimated correlation coefficient between returns on \( X \) and \( S \)
DEFINING THE CONTRACTS

The following table summarizes the payoffs and properties of the various contracts.

Table 1: Foreign stock index contracts as defined by their payoffs at expiration

<table>
<thead>
<tr>
<th>Derivative style</th>
<th>Forward payoff in $</th>
<th>Call option payoff in $</th>
<th>Comments</th>
</tr>
</thead>
</table>
| Foreign-Market   | $X_T(S_T - K)$       | $\max[0, X_T(S_T - K)]$ | • sign of payoff/moneyness is independent of $X_T$
|                  |                      |                          | • magnitude of payoff in $ is independent of $X_T$
| ADR-style        | $X_T S_T - K_S$      | $\max[0, X_T S_T - K_S]$| • sign of payoff/moneyness depends upon $X_T$
|                  |                      |                          | • magnitude of payoff in $ is independent of $X_T$
| GER-style        | $X_0(S_T - K)$       | $\max[0, X_0(S_T - K)]$ | • sign of payoff/moneyness is independent of $X_T$
|                  |                      |                          | • magnitude of payoff in $ is independent of $X_T$
FORWARD CONTRACTS ON FOREIGN STOCK

We begin with the valuation of forward contracts because they are simpler than options, but still capture important features and subtleties involved. A forward contract on a foreign index is an agreement to buy the index on a certain date, at a certain delivery price, in a specified currency. The forward value is the delivery price that makes the forward contract worth zero today.

We will summarize the valuation of forward contracts of three different styles defined in Table 1. Our discussion closely follows the earlier publications on this topic, which are listed under References (see page 21).

Foreign-Market Forwards

We begin with a standard forward contract $f(S, K)$ on the yen-denominated stock worth $S$ today. On the delivery date, an investor long the forward receives $S_T$ yen and pays $K$, so that

$$\text{payoff in yen} = S_T - K \quad (\text{EQ 1})$$

The forward value, $F_S$, in yen is the delivery price that makes the forward contract worth zero today.

You can replicate the payoff $S_T$ by investing today in $e^{-(b + q)(T - t)}$ shares of the stock; by reinvesting both the future dividend yield $q$ on the stock and the future rebate rate $b$, you will end up with exactly one share worth $S_T$ on the delivery date. You can replicate the payment of $K$ yen by selling a zero-coupon bond of face $K$ maturing at time $T$. The current replication value of the forward $f(S, K)$ is

$$f(S, K) = e^{-(b + q)(T - t)}S - e^{-r_Y(T - t)}K \quad (\text{EQ 2})$$

The delivery value $K$ that makes $f(S, K)$ have zero value is the forward value

$$F_S = Se^{(r_Y - b - q)(T - t)}$$

The rate earned on shorting the borrowed stock after paying the rebate is the yen stock loan rate $l_Y$ given by

$$l_Y = r_Y - b \quad (\text{EQ 3})$$

so that the forward value can be written as
The value of the payoff in Equation 1 in dollars is

\[
\text{payoff in dollars} = X_T(S_T - K)
\]

(EQ 5)

where \(X_T\) is the exchange rate at delivery. The current value of the forward contract in dollars is simply its value in yen given by Equation 2 multiplied by the current exchange rate:

\[
f(S, K) = Xe^{-(r_S + q)(T-t)}[e^{(1 - q)(T-t)}S - K]
\]

(EQ 6)

### ADR Forwards

This is a forward contract on the ADR value of the Japanese stock in dollars with payoff given by

\[
\text{payoff in dollars} = X_T S_T - K_S
\]

(EQ 6)

If tradable securities on deposit in the U.S. do exist, the ADR forward can be replicated in the same way as the ordinary forward -- by holding a portfolio that contains:

- a long position in a fraction \(e^{-(b + q)(T-t)}\) of one ADR “share” worth \(XS\);
- a short position in a U.S. zero-coupon bond with face value \(K_S\).

The fair value of the forward contract today is

\[
f(S, K_S) = XS e^{-(b + q)(T-t)} - K_S e^{-r_S(T-t)} \text{ dollars}
\]

The forward price, \(F_{ADR}\), is the delivery price that makes this contract worth zero today, that is

\[
F_{ADR} = XSe^{(l_S - q)(T-t)} \text{ dollars}
\]

(EQ 7)

where \(l_S = r_S - b\) is the ADR’s loan rate.

You may, however, want to create an ADR-like contract on a foreign security which does not have deposits in U.S. In this case, you cannot easily buy a security that has the value \(X_T S_T\) at maturity. Instead, you can replicate the payoff in Equation 6 by investing in the following portfolio:
• a long position in one forward contract on foreign index in yen
• a long position in forward contract to buy $F_S$ yen
• a long position in a U.S. zero-coupon bond with face value $F_S F_X$
• a short position in a U.S. zero-coupon bond with face value $K_S$

The dollar value of this portfolio at time $T$ is

$$X_T (S_T - F_S) + F_S (X_T - F_X) + F_S F_X - K_S = X_T S_T - K_S$$

which is exactly the payoff at delivery we want to replicate. Since the forward contracts in this portfolio are currently worth zero, the value of the whole portfolio today is

$$f(S, K_S) = F_S F_X e^{-r_S (T - t)} - K_S e^{-r_S (T - t)}$$

Again, the forward value of the ADR contract, $F_{ADR}$, is the delivery price that makes this contract worth zero today:

$$F_{ADR} = F_S F_X$$ (EQ 8)

Using the value for $F_S$ given by Equation 4 together with

$$F_X = X e^{(r_S - r_Y) (T - t)}$$ (EQ 9)

and

$$r_S - r_Y = l_S - l_Y$$

we find that the value given by Equation 8 agrees with the previously derived result in Equation 7. It is interesting to note that the forward value of the ADR forward is simply the product of forward price of the index in yen and the forward price of yen in dollars.

Finally, as an alternative, we describe the pricing of ADR forwards using the risk-neutral method. Note that the payoff in Equation 6 is given completely in dollars. To avoid arbitrage, all investments in a dollar-based risk-neutral world should earn the dollar riskless rate. The value of the ADR forward is then the discounted expected value of the ADR forward's payoff at maturity, i.e.,

$$f(S, K_S) = e^{-r_S (T - t)} (E[X_T S_T] - K_S) \text{ dollars}$$ (EQ 10)
Here, \( E[\cdot] \) denotes an expected value at time \( T \) in a dollar-based risk-neutral world. The expected value of index in dollars (whose current value is \( X \)) is

\[
E[X_T S_T] = X S e^{(l_s - q)(T-t)}
\]

where \( q \) is the annual dividend yield. Equation 10 now gives

\[
f(S, K_s) = e^{-r_s(T-t)} (X S e^{(l_s - q)(T-t)} - K_s)
\]

dollars

which leads to the ADR forward price in Equation 7.

**GER Forwards**

The guaranteed exchange rate forward contract has a payoff at expiration given by

\[
\text{payoff in dollars} = X_0 (S_T - K)
\]

where \( X_0 \) is the multiplier that determines how many dollars the contract pays per foreign index point. A contract similar to this one is the Chicago Mercantile Exchange-traded futures contract on the Nikkei 225 index, in which case \( X_0 = 5 \) dollars per yen.

Unfortunately, there is no simple buy and hold strategy that guarantees a payoff of \( X_0 S_T \) dollars at expiration. The reason is that the final dollar value of any static position in Nikkei is unknown, since the dollar value of yen at expiration is unknown. The only strategy for replicating the GER payoff in dollars is a dynamic one, and involves creating a synthetic fund portfolio, \( \Pi \), of traded securities such that it always has the dollar value \( X_0 S \), an exposure of \( X_0 \) dollars to the Nikkei, and zero exposure to the yen. This is achieved by:

- investing \( X_0 S \) dollars;
- borrowing \( \frac{X_0}{X} \) yen; and
- using the borrowed yen to buy \( \frac{X_0}{X} \) shares of Nikkei.

These three components must be adjusted as \( S \) and \( X \) change and as we pay and receive interest and dividends.

The GER forward can be replicated in the same way as the ordinary forward -- by holding a portfolio that contains:
• a long position in a fraction $e^{-(b + q_{syn})(T - t)}$ of one synthetic “share” of the above fund, where $q_{syn}$ is the dividend yield of a share of this fund worth $X_0 S$; and

• a short position in a U.S. zero-coupon bond with face value $X_0 K$.

The fair value of the forward contract today is

$$f(S, K) = X_0 S e^{-(b + q_{syn})(T - t)} - X_0 K e^{-r_s(T - t)} \text{ dollars}$$

It is shown in the appendix that the dividend yield of synthetic security is

$$q_{syn} = q + r_s - r_Y + \sigma_{XS}$$

which implies

$$f(S, K) = X_0 e^{-r_s(T - t)} [Se^{(I_Y - q - \sigma_{XS})(T - t)} - K]$$

The forward price, $F_{GER}$, is the delivery price that makes this contract worth zero today, that is

$$F_{GER} = Se^{(I_Y - q - \sigma_{XS})(T - t)} \text{ yen} \quad (\text{EQ} \ 13)$$

This is similar to the forward value $F_S$ in Equation 4, except that there is an additional term $\sigma_{XS}(T - t)$ in the exponent. $F_{GER}$ in Equation 13 can be written as

$$F_{GER} = Se^{(I_Y - q')(T - t)} \text{ yen} \quad (\text{EQ} \ 14)$$

where

$$q' = q + \sigma_{XS} = q + \rho_{XS} \sigma_X \sigma_S \quad (\text{EQ} \ 15)$$

is an “effective dividend” comprised of the dividend yield $q$ and an additional “induced dividend yield” $\sigma_{XS}$. This shows that a positive correlation $\rho_{XS}$ increases the effective dividend and in turn lowers the GER forward value relative to an ordinary forward contract. We will present an intuitive explanation of this effect below after we summarize the valuation of a GER forward contract using the risk-neutral method.
In a risk-neutral world, the value of the GER forward is the discounted expected value of the payoff at time $T$:

$$f(S, K) = e^{-r_s(T-t)}(E[X_0S_T] - X_0K) \text{ dollars} \tag{EQ 16}$$

The value of $E[X_0S_T]$ cannot be calculated directly because $X_0S_T$ is not the price of tradable security. However, we can use the following mathematical result:

If $S_T$ and $X_T$ are lognormally distributed random variables with return covariance $\sigma_{XS}$ then at time $t$:

$$E[X_TS_T] = E[X_T]E[S_T]e^{\sigma_{XS}(T-t)} \tag{EQ 17}$$

Using the result in Equation 11 together with

$$E[X_T] = X e^{(r_s-r_Y)(T-t)} \tag{EQ 18}$$

we find that

$$E[S_T] = Se^{(l_Y - q - \sigma_{XS})(T-t)} \tag{EQ 19}$$

Equation 16 now gives

$$f(S, K) = e^{-r_s(T-t)}X_0(Se^{(l_Y - q - \sigma_{XS})(T-t)} - K) \text{ dollars}$$

from which we read off the GER forward value as given in Equation 13.

An intuitive explanation of the effect of correlation goes as follows. Suppose that over a short time period all securities can either move up or down. Consider the extreme case where Nikkei index $S$ and the dollar value of yen $X$ are 100% positively correlated, that is $\rho_{XS} = 1$. This means that an up (down) move in the index is theoretically always accompanied by an up (down) move in the yen. In the following table, we illustrate their evolution together with the corresponding evolution of a GER version of the forward contract, $G$, with the guaranteed exchange rate set to the initial dollar/yen rate, $X_0 = X$. 

To highlight the essence of the argument, we assume that all securities have zero interest and dividends over this short time period. This means that the forward value of the standard contract equals the index value. Suppose that the initial dollar value of GER forward is also SX and consider an initial investment in the following hedged portfolio, \( \Pi \):

- a long position in one GER forward contract with \( X_0 = X \)
- a short position in one share of Nikkei index converted to dollars
- a long position in \( S \) yen

The initial value of this portfolio is

\[
\Pi_i = SX - SX + SX = SX
\]

The final value of the portfolio in the up state is

\[
\Pi_f^{up} = S_uX - S_uX_u + S X_u
\]

and in the down state

\[
\Pi_f^{down} = S_dX - S_dX_d + S X_d
\]

The net P\&L after the up move is

\[
\Pi_f^{up} - \Pi_i = -(S_u - S)(X_u - X) < 0
\]

and after the down move

\[
\Pi_f^{down} - \Pi_i = -(S_d - S)(X - X_d) < 0
\]
where we used the obvious relation $S_U > S > S_d$. If the net P&L is negative for both up and down moves in the index, someone taking the opposite position from the one in our portfolio can theoretically earn riskless profit. Therefore, we conclude that the fair initial value of GER forward should be less than the assumed value $S_X$. This is equivalent to saying that the GER forward will appreciate more slowly which, in turn, means that the effective dividend is higher than the regular dividend. In short, positive correlation implies a positive dividend. Similar arguments can be used to show that negative correlation leads to negative effective dividend and to a higher GER forward price.

**Summary of Results for Forwards**

Table 3 summarizes the main result for various types of forward contracts considered here.

**Table 3: Summary of results for forward contracts**

<table>
<thead>
<tr>
<th>Style</th>
<th>Payoff in $</th>
<th>Forward price</th>
<th>Current value of forward in $</th>
</tr>
</thead>
<tbody>
<tr>
<td>Foreign-Market</td>
<td>$X_T(S_T - K)$</td>
<td>$S e^{(l_Y - q)(T - t)}$ yen</td>
<td>$X e^{-r_Y (T - t)} [S e^{(l_Y - q)(T - t)} - K]$</td>
</tr>
<tr>
<td>ADR-style</td>
<td>$X_T S_T - K_S$</td>
<td>$X S e^{(l_s - q)(T - t)}$ $</td>
<td>$</td>
</tr>
<tr>
<td>GER-style</td>
<td>$X_0(S_T - K)$</td>
<td>$S e^{(l_Y - q - \sigma_X S)(T - t)}$ yen</td>
<td>$X_0 e^{-r_s (T - t)} [S e^{(l_Y - q - \sigma_X S)(T - t)} - K]$</td>
</tr>
</tbody>
</table>
We now turn our discussion to options. For European options, we can write analytic expressions assuming constant volatility and interest rates. Although not realistic, the results in this simplified world will help us understand generic properties of options on foreign indexes and strategies for their hedging. To be specific, we will discuss call options. The results for put options can be derived in a similar way.

**Foreign-Market Options**

We first consider standard options which are traded on Japanese exchanges and which can be hedged by shorting the foreign index. Therefore, they can be valued by a risk-neutral method. The payoff is given by

\[ C(T) = \max[0, X_T(S_T - K)] \]  

and the present value of the call option in dollars is simply the option’s value in yen converted to dollars at the current exchange rate

\[ C = X e^{-r(T-t)} [F_S N(d_1) - KN(d_2)] = XC_Y \]

Here \( F_S \) is the forward value of the index (see Equation 4),

\[ F_S = Se^{(r_Y - q)(T - t)} \]

and

\[ d_1 = \frac{\log(S/K) + (r_Y - q + \frac{\sigma_S^2}{2})(T - t)}{\sigma_S \sqrt{T - t}} \]

\[ d_2 = d_1 - \sigma_S \sqrt{T - t} \]

The price of a call option depends on the Japanese discount and stock loan rate, the index volatility and dividend yield. These parameters affect the price in the same way as in the case of an ordinary option. In addition, the price is proportional to the current exchange rate. The correlation between \( S \) and \( X \) is irrelevant.

A long call option can be hedged in the Japanese market in the same way as an ordinary option -- by shorting \( n_S = \Delta_Y \) shares of Nikkei index where
The dollar value of the hedged position is
\[ XC_Y - X(\Delta_Y S) = X(C_Y - \Delta_Y S) \]
which clearly depends on the exchange rate. The exchange rate risk can be hedged by shorting
\[ n_X = C_Y - \Delta_Y S \text{ yen} \]
It is easy to see that a portfolio consisting of a call option and short positions in \( \Delta_Y \) shares of Nikkei and \( n_X \) yen
\[ \Pi = C - n_S\{XS\} - n_X\{X\} \]
is insensitive to small changes in \( S \) and \( X \).

**ADR Options**

The dollar payoff for these options is
\[ C_{ADRs}(T) = \max[0, X_T S_T - K_S] \]
This is just a U.S.-traded option on the foreign index denominated in dollars that can be hedged by shorting the index and converting payment to dollars. This can be thought of as an option with strike price \( K_S \) on an asset \( XS \) which is denominated in dollars, pays a continuous dividend yield \( q \) and has volatility
\[ \sigma = \sqrt{\sigma_S^2 + \sigma_X^2 + 2 \rho_{XS} \sigma_S \sigma_X} \]
The option value in dollars is given by Black-Scholes formula with the adjusted volatility:
\[ C_{ADRs} = e^{-r_s(T-t)} \left[ F_{ADRs} N(d_1) - K_S N(d_2) \right] \quad \text{(EQ 22)} \]
where
\[ F_{ADRs} = X S e^{(l_s - q)(T-t)} \]
is the ADR forward value, and
The value of the ADR option is independent of the Japanese discount and stock loan rates. The effects of index price, current exchange rate, the U.S. discount and stock loan rates and time to maturity are as expected. The value of the ADR option increases with increasing overall volatility \( \sigma \). The effects of individual volatilities are no longer clear-cut. If \( S \) and \( X \) are positively correlated, then the volatilities affect the option's value in a conventional way. In the case of sufficiently negative correlation, it is possible that increasing individual volatilities may cause a decrease in option price. Specifically, for a negative \( \rho_{XS} \), when \( \rho_{XS} \sigma_X > \sigma_S \), the value of the ADR option decreases as the index volatility increases. However, since the exchange rate volatility is usually smaller than index volatility, this situation does not happen very often. Similarly, for a negative \( \rho_{XS} \), when \( \rho_{XS} \sigma_S > \sigma_X \), the value of the ADR option increases as the index volatility increases.

Theoretically this option has no currency risk and a long call should be hedged by shorting

\[
d_1 = \frac{\log \left( \frac{X}{K_S} \right) + (r_s - q + \frac{\sigma^2}{2})(T-t)}{\sigma \sqrt{T-t}}
\]

\[
d_2 = d_1 - \sigma \sqrt{T-t}
\]

\[\Delta = \frac{\partial C_{ADR}}{\partial (X_S)} = e^{-(q + b)(T-t)} N(d_1)\]

shares of Nikkei. No holding of yen is necessary, since the index held also hedges the exchange rate risk.

**GER (Quanto) Options**

GER or quanto options have payoff

\[C_{GER}(T) = \max [0, X_0(S_T - K)]\]

The difficulty in pricing this option comes from the fact that there is no traded security whose dollar value is \( X_0S \) at any time \( t \). Since you cannot hedge directly with the underlyer, you cannot price by arbitrage. As we described in the discussion of valuing a GER forward, it is possible to construct a portfolio, \( \Pi \), of traded securities such that it always has the value \( X_0S \).
The GER option can then be understood as an ordinary option with strike \( K X_0 \) dollars on an imaginary stock whose price distribution is that of \( X_0 S_T \) with volatility \( \sigma_S \) and mean growth rate \( l_Y - q - \sigma_X S \) (see the discussion of GER forwards). The call option price is now simply

\[
C_{\text{GER}} = X_0 e^{-r_s(T-t)} [F_{\text{GER}} N(d_1) - KN(d_2)] \tag{EQ 23}
\]

where

\[
F_{\text{GER}} = S e^{(l_Y - q - \sigma_X S)(T-t)}
\]

is the GER forward value, and

\[
d_1 = \frac{\log \left( \frac{S}{K} \right) + \left( l_Y - q - \sigma_X S + \frac{\sigma_S^2}{2} \right)(T-t)}{\sigma_S \sqrt{T-t}}
\]

\[
d_2 = d_1 - \sigma_S \sqrt{T-t}
\]

There are three interesting features of this result:

- The value of the GER option depends on both the U.S. discount rate and the Japanese stock loan rate. The Japanese stock loan rate affects the option's price in a conventional way, while increasing the U.S. discount rate decreases the option price.

- The value of the GER option does not depend on the prevailing exchange rate. Instead, it depends on the correlation between the index level and the exchange rate. Note that even in the case of zero correlations, the price of the GER option is not exactly equal to the price of an ordinary option, since the rates in two different currencies are involved.

- Increasing exchange rate volatility increases the call option value if the correlation between \( X \) and \( S \) is negative, and decreases the option value if the correlation is positive. The effect of change in index volatility depends, among other things, on the sign and size of the correlation coefficient and the degree to which the option is in the money. (Typically, only when the option is deep-in-the-money, increasing the index volatility will cause the price to decrease.)
In order to understand how to hedge a GER option, it is useful to write Equation 23 in the following form

\[ C_{\text{GER}} = X_0 e^{(r_Y - r_s)(T - t)} C_Y. \]  

(EQ 24)

where \( C_Y \) is the price of a standard option on the Nikkei in yen, but with an effective dividend yield

\[ q' = q + \rho X S \sigma_X \sigma_S. \]

Since the price of a GER option does not depend on the exchange rate, you may naively think that you only need to hedge the risk associated with the index price. This can be done in the Japanese market by shorting \( n_S \) shares of the index where

\[ n_S = \frac{X_0}{X} e^{(r_Y - r_s) t} \Delta_Y, \]

and

\[ \Delta_Y = \frac{\partial C_Y}{\partial S} \]

is the option’s delta in the Japanese market with a modified dividend yield. Although the price does not depend on the exchange rate, holding \( n_S \) Nikkei shares in your hedging portfolio introduces currency risk, which can be offset by going long an equivalent amount of yen, i.e.,

\[ n_X = n_S S \text{ yen} \]

Thus, as you adjust your delta-hedge of the option you need to change your long position in yen depending on the number of shares in your hedge and the prevailing index price.

One can check that a portfolio consisting of a GER call option, a short position in \( n_S \) shares of Nikkei and a long position in \( n_X \) yen

\[ \Pi = C_{\text{GER}} - n_S \{X S\} + n_X \{X\} \]

is insensitive to small changes in \( S \) and \( X \).
In Table 4, we summarize the dependence of option prices on various parameters.

**Table 4: Dependence of option prices on various parameters**

<table>
<thead>
<tr>
<th>Style</th>
<th>$r_Y$</th>
<th>$l_Y$</th>
<th>$r_s$</th>
<th>$l_s$</th>
<th>$q$</th>
<th>$X$</th>
<th>$\sigma_S$</th>
<th>$\sigma_X$</th>
<th>$\rho_{XS}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Foreign-Market</td>
<td>yes</td>
<td>yes</td>
<td>no</td>
<td>no</td>
<td>yes</td>
<td>yes</td>
<td>yes</td>
<td>yes</td>
<td>no</td>
</tr>
<tr>
<td>ADR-style</td>
<td>no</td>
<td>no</td>
<td>yes</td>
<td>yes</td>
<td>yes</td>
<td>yes</td>
<td>yes</td>
<td>yes</td>
<td>yes</td>
</tr>
<tr>
<td>GER-style</td>
<td>no</td>
<td>yes</td>
<td>yes</td>
<td>no</td>
<td>yes</td>
<td>no</td>
<td>yes</td>
<td>yes</td>
<td>yes</td>
</tr>
</tbody>
</table>

Using the rules summarized in Table 5, you can find the price of options on a foreign index, even if you only have a standard Black-Scholes calculator that allows you to input the current price of the underlyer, strike, discount rate, loan rate, dividend yield and volatility.

**Table 5: How to use a Black-Scholes calculator to price options on foreign index**

<table>
<thead>
<tr>
<th>Style</th>
<th>underlyer</th>
<th>strike</th>
<th>discount rate</th>
<th>loan rate</th>
<th>dividend yield</th>
<th>volatility</th>
</tr>
</thead>
<tbody>
<tr>
<td>Foreign-Market</td>
<td>$XS$</td>
<td>$XK$</td>
<td>$r_Y$</td>
<td>$l_Y$</td>
<td>$q$</td>
<td>$\sigma_S$</td>
</tr>
<tr>
<td>ADR-style</td>
<td>$XS$</td>
<td>$K_S$</td>
<td>$r_s$</td>
<td>$l_s$</td>
<td>$q$</td>
<td>$\sqrt{\sigma_S^2 + \sigma_X^2 + 2\rho_{XS}\sigma_S\sigma_X}$</td>
</tr>
<tr>
<td>GER-style</td>
<td>$X_0S$</td>
<td>$X_0K$</td>
<td>$r_s$</td>
<td>$l_Y$</td>
<td>$q + \rho_{XS}\sigma_X\sigma_S$</td>
<td>$\sigma_S$</td>
</tr>
</tbody>
</table>

Note that the parameters specified in this table can be used to build the binomial tree which you can use to price American options.
Finally, we summarize the rules for delta hedging a long call position in Table 6:

<table>
<thead>
<tr>
<th>Style</th>
<th>Number of shares</th>
<th>Amount of yen</th>
</tr>
</thead>
<tbody>
<tr>
<td>Foreign-Market</td>
<td>short $\Delta_Y$ shares</td>
<td>short $C_Y - \Delta_Y S$ yen</td>
</tr>
<tr>
<td></td>
<td>$\Delta_Y = \text{delta of the option in Japanese market}$</td>
<td>$C_Y = \text{option price in Japanese market}$</td>
</tr>
<tr>
<td></td>
<td>$\Delta_Y = \text{delta of the option in Japanese market}$</td>
<td>$S = \text{current index level}$</td>
</tr>
<tr>
<td>ADR-style</td>
<td>short $\Delta$ ADR shares</td>
<td>none</td>
</tr>
<tr>
<td></td>
<td>$\Delta = \text{delta of the option on ADR with modified volatility}$</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$\sigma = \sqrt{\frac{2}{\sigma_S^2 + \sigma_X^2 + 2\rho_{XS}\sigma_S\sigma_X}}$</td>
<td></td>
</tr>
<tr>
<td>GER-style</td>
<td>short $n_S$ shares</td>
<td>long $n_S S$ yen</td>
</tr>
<tr>
<td></td>
<td>$\Delta_{Y'} = \text{option’s delta in Japanese market with a modified dividend yield } q' = q + \rho_{XS}\sigma_X\sigma_S$</td>
<td>$n_S = \text{number of shares}$</td>
</tr>
<tr>
<td></td>
<td>$S = \text{current index level}$</td>
<td>$S$</td>
</tr>
</tbody>
</table>
APPENDIX

In this Appendix, we derive the effective dividend yield on a “share” of a portfolio which at any time has value $X_0S$ dollars. As discussed in the text, such a portfolio consists of the following three components:

- investing $n_S = X_0S$ dollars
- borrowing $n_Y = \frac{X_0}{X}$ yen
- using the borrowed yen to buy $n_S = \frac{X_0}{X}$ shares of Nikkei.

At any time, the dollar value of this portfolio is

$$\Pi = n_S + n_Y X + n_S S X$$

The change in value of one synthetic “share” of this portfolio during a small time period $\Delta t$ comes from a change in the exchange rate $\Delta X$ and a change in the foreign index price $\Delta S$, as well as the interest earned and paid:

- interest earned on invested dollars: $n_S r_S \Delta t$
- interest paid on borrowed yen converted to dollars: $(n_Y r_Y \Delta t) X$
- the profit/loss from borrowed yen: $n_Y \Delta X$
- the change in value of Nikkei shares in dollars: $n_S \Delta (XS) = n_S (X\Delta S + S\Delta X + \Delta X \Delta S)$
- the dividend earned on Nikkei shares in dollars: $(n_S qS \Delta t) X$

The total change is simply the sum of all these contributions. Taking into account the amounts of each of the portfolio components, this can be written as

$$\Delta \Pi = X_0 \Delta S + X_0 S \left( q + r_s - r_Y + \frac{1}{\Delta t} \frac{\Delta X \Delta S}{X S} \right) \Delta t$$

The first term of this total change is due to the change in price of the synthetic share. The second term is the dividend yield. If we assume that the returns of the yen value in dollars and of the Nikkei price in yen are jointly normally distributed and correlated, as the time interval $\Delta t$ approaches zero we have

$$\frac{1}{\Delta t} \frac{\Delta X \Delta S}{X S} = \rho X S \sigma_X \sigma_S$$
where $\sigma_X$ and $\sigma_S$ are the annualized volatilities of returns and $\rho_{XS}$ is the correlation coefficient of these two returns. Putting this all together, we find the dividend yield of one synthetic share to be

$$q_{\text{syn}} = q + r_S - r_Y + \rho_{XS} \sigma_X \sigma_S$$
REFERENCES


SELECTION QUANTITATIVE STRATEGIES PUBLICATIONS

June 1990  Understanding Guaranteed Exchange-Rate Contracts In Foreign Stock Investments  
Emanuel Derman, Piotr Karasinski and Jeffrey Wecker

Jan. 1992  Valuing and Hedging Outperformance Options  
Emanuel Derman

Mar. 1992  Pay-On-Exercise Options  
Emanuel Derman and Iraj Kani

June 1993  The Ins and Outs of Barrier Options  
Emanuel Derman and Iraj Kani

Jan. 1994  The Volatility Smile and Its Implied Tree  
Emanuel Derman and Iraj Kani

May 1994  Static Options Replication  
Emanuel Derman, Deniz Ergener and Iraj Kani

May 1995  Enhanced Numerical Methods for Options with Barriers with Barriers  
Emanuel Derman, Iraj Kani, Deniz Ergener and Indrajit Bardhan

Dec. 1995  The Local Volatility Surface: Unlocking the Information in Index Option Prices  
Emanuel Derman, Iraj Kani and Joseph Z. Zou

Feb. 1996  Implied Trinomial Trees of the Volatility Smile  
Emanuel Derman, Iraj Kani and Neil Chriss

Apr. 1996  Model Risk  
Emanuel Derman

Aug. 1996  Trading and Hedging Local Volatility  
Iraj Kani, Emanuel Derman and Michael Kamal

Apr. 1997  Is the Volatility Skew Fair?  
Emanuel Derman, Michael Kamal, Iraj Kani and Joseph Zou
Apr. 1997  Stochastic Implied Trees: Arbitrage Pricing with Stochastic Term and Strike Structure of Volatility  
Emanuel Derman and Iraj Kani

Sept. 1997  The Patterns of Change in Implied Index Volatilities  
Michael Kamal and Emanuel Derman

Nov. 1997  Predicting the Response of Implied Volatility to Large Index Moves: An October 1997 S&P Case Study  
Emanuel Derman and Joe Zou